Gamma-ray Emission Properties from Mature Pulsars in the Galaxy and in the Gould Belt

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ABSTRACT

We study the γ-ray emission properties of pulsars by using a new self-consistent outer gap model. The outer gap can exist in pulsars with age over million years old if the effect of magnetic inclination angle as well as the average properties of the outer gap are considered. The mature γ-ray pulsars, whose ages are between 0.3 to 3 million years old, are able to move up to high galactic latitude. Moreover, their γ-ray luminosity are weaker and their spectra are softer than those of younger γ-ray pulsars in the galactic plane significantly. We use a Monte Carlo method to simulate the statistical properties of γ-ray pulsars in the Galaxy as well as in the Gould Belt. We find that γ-ray pulsars located at $\mid b \mid < 5^\circ$ and located at $\mid b \mid > 5^\circ$ have very different properties. High galactic latitude γ-ray pulsars are dominated by mature pulsars with longer periods, weaker fluxes and softer spectra. If the pulsar birth rate in the Galaxy and the Gould Belt are $\sim 10^{-2} yr^{-1}$ and $\sim 2 \times 10^{-5} yr^{-1}$ respectively, there are 42 and 35 radio-quiet γ-ray pulsars for $\mid b \mid < 5^\circ$ and $\mid b \mid > 5^\circ$ respectively. Radio-quiet γ-ray pulsars from the Gould Belt are 2 and 13 for $\mid b \mid < 5^\circ$ and $\mid b \mid > 5^\circ$ respectively. We suggest that a good fraction of unidentified EGRET γ-ray sources may be these radio-quiet γ-ray pulsars. Furthermore γ-ray pulsars located at $\mid b \mid > 5^\circ$ satisfies $L_\gamma \propto L_{sd}^{\beta}$ whereas $L_\gamma \propto L_{sd}^{\delta}$ for γ-ray pulsars in the galactic plane, where $\beta \sim 0.6$ and $\delta \sim 0.3$ respectively.

Subject headings: gamma-rays: theory - pulsars: general - stars: neutron - stars: statistics

1. Introduction

There are 170 unidentified γ-ray sources in the third EGRET catalog (Hartman et al. 1999), where $\sim$50 sources close to the Galactic plane with $|b| < 5^\circ$ and $\sim$70 sources in

the medium latitudes with $|b|$ between $5°$ and $30°$. For those unidentified γ-ray sources in the Galactic plane, many of them are associated with Wolf-Rayet and Of stars, SNRs and OB stars(Montmerle 1979; Kaaret & Cottam 1996; Yadigaroglu & Romani 1997, Romero et al. 1999). Most of these objects are considered as pulsar tracers, therefore it is natural to suggest that these low latitude sources may be Geminga-like pulsars, which are radio-quiet pulsars (Yadigaroglu & Romani 1995; Cheng & Zhang 1998; Zhang, Zhang & Cheng 2000). Since γ-ray pulsars are known to be steady γ-ray emitters, it has been suggested that the variability should be a good indicator to identify the real pulsar candidates from the unidentified EGRET sources (Romero, Combi & Colomb 1994; Mclaughlin et al. 1996; Zhang et al. 2000; Torres et al. 2001; Nolan et al. 2003). Gehrels et al. (2000) define a class of sources as steady: a source is steady if the most significant detection of a source in 3EG catalogue is for a timescale of years and if that particular flux is within 3σ of the flux calculated for the full data set. From this classification, 48 unidentified sources at $|b| < 5°$ and 72 unidentified sources at $|b| > 5°$ are steady sources respectively. On the other hand, Grenier (2001) suggests another definition called persistent sources, which are those detected sources with a significance $\sqrt{TS} > 4$ for every observation at $|b| > 2.5°$. If we assume that the sources with a significance $\sqrt{TS} > 5$ at $|b| \leq 2.5°$ are persistent sources, there are 40 (45) persistent unidentified EGRET sources at $|b| < 5°$ ($|b| > 5°$).

However, the spectral properties of medium latitude sources are significantly softer, fainter and have a steeper logN-logS function than those at low latitudes (Gehrels et al. 2000). It has been suggested that they are associated with recent supernovae in the nearby Gould Belt (Grenier 1997; Gehrels et al. 2000; Grenier 2000). Their natures remain as mystery. Harding and Zhang (2001) used the polar cap models (Daugherty & Harding 1996; Harding & Muslimov 1998) to investigate whether γ-ray pulsars viewed at a large angle to the neutron star magnetic pole could contribute to unidentified EGRET sources in the medium latitudes associated with the Gould Belt. They suggest that the off-beam γ-rays come from high-altitude curvature emission of primary particles can radiate over a large solid angle and have a much softer spectrum than that of the main beams, and at least some of radio-quiet Gould Belt sources detected by EGRET could be such off-beam γ-ray pulsars.

Recently, the brightest of unidentified EGRET sources in the medium latitude, 3EG J1835+5918, is strongly suggested as the second Geminga-like pulsar (radio-quiet pulsar) by multi-wavelength observations including Chandra, HST and Jodrell Bank (Mirabal et al. 2000; Mirabal & Halpern 2001; Halpern et al. 2002). The X-ray spectrum can be described by two components: a soft thermal X-ray spectrum with a characteristic temperature ($T_{\rm inf} \approx 3 \times 10^5$K and a power law hard tail with a photon index $\gamma \approx$2, which closely resembles to the X-ray spectrum of Geminga. The repeated radio observations at Jodrell Bank did not show any periodicity. The X-ray data suggests that the distance is between 250-800pc, which is

consistent with the distance to the Gould Belt.

In this paper, we use the revised outer gap models (Zhang et al. 2004) to investigate emission properties of γ-ray pulsars in the Galaxy as well as in the Gould Belt. This revised outer gap model is based on the original outer gap models (Cheng, Ho & Ruderman 1986a, **hereafter CHR I**, 1986b; Zhang & Cheng 1997) but takes into account the effect of the inclination angle (α), which is the angle between the magnetic axis and the rotation axis, in determining the size of the outer gap,which is defined as the ratio between the dimension of the gap perpendicular to the magnetic field and the light cylinder radius. The revised model also takes into account the fact that the typical radiation region of the outer gap is not necessary at half of the light cylinder instead it should be better represented by an appropriate average over the entire outer gap. This effect is particular important for old pulsars. If the outer gap is only represented at half of the light cylinder, then the outer gap is assumed to be turn off when the gap size at this region is larger than unity. This new model has taken the entire active region of the outer gap into account. As long as the gap size is less than unity in some parts of magnetosphere, the outer gap still exist. This effect allows some pulsars with appropriate combination of α, P and B, can maintain their outer gaps until a few million years old. These pulsars are able to move up to high galactic latitude and their ages make them weak γ-ray sources. Furthermore, we will show that these pulsars will emit softer spectra and their relation between γ-ray luminosity and spin-down power differs from that of younger pulsars located in the galactic plane. We organize the paper as follows. In section 2, we review the new outer gap models. In section 3, we study the γ-ray emission properties of pulsars by using the new self-consistent outer gap model. In section 4, we describe a Monte Carlo simulation method to determine the statistical distributions of radio-quiet and radio-loud γ-ray pulsars. In section 5, we summarize the simulation results and discuss their implications. Finally, a brief conclusion is given in section 6.

2. Outer Gap Models

Zhang & Cheng (1997) have proposed a self-consistent mechanism to describe the high-energy radiation from the rotation-powered pulsars. In this model, the radiation mechanism of relativistic charged particles from a thick outermagnetospheric accelerator (outer gap) is synchro-curvature radiation (Cheng & Zhang 1996) and the characteristic energy of high-energy photon emitted from the outer gap is determined by the pulsar global parameters, i.e. rotation period P and the dipolar magnetic field B, as well as the fractional size of the outer gap (f), which is a ratio between the mean vertical separation of the outer gap boundaries in the plane of the rotation axis and the magnetic axis to the light cylinder radius, and is

given by

$$E_\gamma(f) \approx 5.0 \times 10^7 f^{3/2} B_{12}^{3/4} P^{-7/4} \left(\frac{r}{R_L}\right)^{-13/8} eV \tag{1}$$

where B_{12} is the dipolar magnetic field in unites of 10^{12}G, R_L is the light cylinder radius and r is the distance to the neutron star. The γ-ray spectrum becomes exponentially decay beyond $E_\gamma(f)$. The fractional size of the outer gap determines the total γ-ray luminosity from the outer gap (Zhang & Cheng 1997) and is given by

$$L_\gamma = f^3 L_{sd} \tag{2}$$

where $L_{sd} = 3.8 \times 10^{31} B_{12}^2 P^{-4} ergs^{-1}$ is the spin-down power of pulsar. CHRI has shown that the inner boundary of the static outer gap begins at the null charge surface($\mathbf{\Omega} \cdot \mathbf{B} = 0$). Recently Hirotani & Shibata (2001) has shown that if there is injected current from the inner boundary, the position of the inner boundary can shift either toward the star or even close to the light cylinder depending on the sign and the magnitude of the injected current. However, it is not clear what causes this injected current. In this paper we shall assume that the outer gap begins at the null charge surface. The radial distance from the null surface surface to the star is r_{in}, which is a function of the magnetic inclination angle α and is given by $\frac{r_{in}}{R_L} = \frac{\sin^2(\theta_{in}-\alpha)}{\sin\theta_c \sin^2(\theta_c-\alpha)}$, where θ_{in} is the polar angle between of r_{in} determined by $\tan\theta_{in} = \frac{1}{2}(3\tan\alpha + \sqrt{9\tan^2\alpha + 8})$ and θ_c is the polar angle to the position where the first open field line intercepts the light cylinder determined by $\tan\theta_c = -\frac{3}{4\tan\alpha}(1+(1+8\tan^2\alpha/9)^{1/2})$. The fractional size of the outer gap is limited by the pair production between the soft thermal X-rays with characteristic energy E_x from the stellar surface and the high-energy photons with energy $E_\gamma(f_o)$ emitted from the outer gap. The energy of the soft X-ray photons is determined by the backflow of the primary electrons/positrons. Each of these backflow particles can still maintain about $10.6\mathrm{P}^{1/3}$ ergs and deposit on the stellar surface. This energy will be emitted as soft thermal X-rays from the stellar surface (Halpern & Ruderman 1993), whose characteristic energy is given by $E_x(f_o) \approx 1.2 \times 10^2 f_o^{1/4} B_{12}^{1/4} P^{-5/12}$eV. Although the thermal X-ray photon density is low, every pair resulting from X-ray and high-energy photon interactions can emit $\sim 10^5$ high-energy photons when they are accelerated in the gap. Such a large multiplicity can produce sufficient number of $e^\pm$ pairs as to sustain the outer gap. Assuming $r = R_L/2$ and head-on collision, they obtained the fractional size of the outer gap as

$$f_o(B,P) \approx 5.5 P^{26/21} B_{12}^{-4/7} \tag{3}$$

from the condition of photon-photon pair production $E_x E_\gamma(1-\cos(\theta_{X\gamma})) = 2(m_e c^2)^2$.

However, this model did not take into account the fact that when the magnetic inclination angle (α) becomes large, the characteristic energy of high-energy photons from the

gap, which depends on α, also increases. In fact both E_x and $\theta_{X\gamma}$ also depend on α. Most important, Zhang and Cheng (1997) has assumed a typical distance ($r = \frac{R_L}{2}$) to represent the outer gap. In other words, most radiation and pair production activities are assumed to take place at this characteristic region. They assume that when $f_o(r = \frac{R_L}{2}, B, P) > 1$. the outer gap does not exist. However, even the mid-distance to the outer gap depends on the inclination angle because the outer gap begins at the null charge surface, which depends on α. Realistically, as long as $e^{\pm}$ pairs can be produced beyond the null surface, the outer gap should be still active, namely, accelerating charged particles. Of course, for regions with $f(r) > 1$. the gap potential should drop off rapidly and hence becomes unimportant. Therefore the active region of the outer gap should begin at null surface and stop at a distance r_b where $f(r_b) = 1$. (Zhang et al. 2004). In explaining the detail γ-ray spectrum of a given pulsar, it is important to know the radiation coming from which parts of the pulsar magnetosphere (Cheng, Ruderman & Zhang 2000). However, in order to study the statistical properties of pulsars a representative region is a very useful approximation. They assume that the representative region of the outer gap is the averaging distance to the gap, they obtain the mean fractional size of the gap $f(P, B, \alpha)$, which can be approximately expressed as

$$f(\alpha, B, P) \approx f_o(B, P)\eta(\alpha, B, P) \tag{4}$$

where $\eta(\alpha, B, P)$ is a monotonically function of α, B and P. It roughly decreases by a factor 3 from young pulsars with large inclination angle to old pulsars with small inclination angle. The integral expression of $f(\alpha, B, P)$ is given by Eq. (38) and the variation of η for different pulsars is given in figure 3 of Zhang et al. (2004) respectively. Although the variation of η is only a factor of 3, the implications are very important. First the cut-off period of γ-ray pulsars is about a factor of $3^{21/26} = 2.4$ longer for fixed α and B in comparing with the old model. Secondly the cut-off age of γ-ray pulsars is about a factor of 5.8 longer as well. This means that there are a lot more γ-ray pulsars in high galactic latitude than previously expected. These γ-ray pulsars with older age can move up to the high galactic latitude and may contribute to the unidentified EGRET sources.

3. Gamma-ray Emission Properties of Mature Pulsars

For the thick outer gap (Zhang & Cheng 1997), γ-rays are produced inside the thick outer gap by curvature radiation from the primary $e^{\pm}$ pairs along the curved magnetic field lines. However, Cheng & Zhang (1996) studied the radiation from the charged particles in the curved magnetic field, and pointed out that the radiation should be described more accurately by a general radiation mechanism called synchro-curvature radiation mechanism, in which the radiation is being emitted by the charged particles moving in a spiral trajectory

along the curved magnetic field lines. This mechanism differs from synchrotron and curvature mechanisms in general, but reduces to either synchrotron radiation when the radius of curvature of the local magnetic field lines is infinite or to curvature radiation when the pitch angle is zero. In fact, when the synchrotron gyro-radius $r_B = \gamma mc^2 \sin\theta_p/eB(r)$ and the curvature radius of field $s \approx \sqrt{rR_L}$ is comparable, the synchro-curvature mechanism really provides a significant improvement, where γ is the Lorentz factor of the accelerated particles and θ_p is the pitch angle of the charged particles in the curved magnetic field. Zhang & Cheng (1997) used this mechanism to describe the production of non-thermal photons from the primary $e^{\pm}$ pairs along the curved magnetic field lines in the outer gap. The primary $e^{\pm}$ pairs have an approximate power-law distribution inside the outer gap because the energy and density of the primary $e^{\pm}$ pairs depend on local values of magnetic field, electric field and radius of curvature. In steady state, the energy distribution of the accelerated particles in the outer gap is $(dN/dE_e) \propto E_e^{-16/3}$, where E_e is the energy of the accelerated particle. Using the general formula of the synchro-curvature radiation power spectrum given by Cheng & Zhang (1996) and $(dN/dE_e)dE_e = (dN/dx)dx$, where $x = s/rR_L$, the differential flux at the Earth is (Zhang & Cheng 1997)

$$F(E_\gamma) \approx \frac{1}{\Delta\Omega d^2}\frac{\dot{N}_0}{E_\gamma}\int_{x_{min}}^{x_{max}} x^{3/2}\frac{R_L}{R_c}\left[\left(1+\frac{1}{R_c^2Q_2^2}\right)F(y)-\left(1-\frac{1}{R_c^2Q_2^2}\right)yK_{2/3}(y)\right]dx \quad , \quad (5)$$

where $\Delta\Omega$ is the solid angle of γ-ray beaming, d is the distance to the pulsar, $\dot{N}_0 = \sqrt{3}e^2\gamma_0 N_0/hR_L$, $N_0 \approx 1.4\times 10^{30} f(B_{12}/P)R_6^3$, $\gamma_0 \approx 2\times 10^7 f^{1/2}(B_{12}/P)^{1/4}$, $R_c = xR_L/[(1 + r_B/(xR_L))\cos^2\theta_p + (R_L/r_B)x\sin^2\theta_p]$, $Q_2 = (1/xR_L)[((r_B/xR_L) + 1 - 3(R_L/r_B)x)\cos^4\theta_p + 3(R_L/r_B)x\cos^2\theta_p + (R_L/r_B)^2x^2\ \sin^4\theta_p]^{1/2}$, and $\sin\theta_p \approx 0.79 f^{1/2}B_{12}^{-3/4}P^{7/4}x^{17/4}$; $F(y) = \int_y^\infty K_{5/3}(z)dz$, where $K_{5/3}$ is the modified Bessel functions of order 5/3, $y = E_\gamma/E_c$ and $E_c = (3/2)(\hbar c\gamma^3/x)(xQ_2)$ is the characteristic energy of the synchro-curvature photons. x_{min} and x_{max} are the minimum and maximum values of x. For x_{min}, it can estimated as $x_{min} = (r_{in}/R_L)^{1/2}$. The maximum value of x can be estimated by $x_{max} \approx \frac{\theta_p}{\theta_f}\sqrt{r_f/R_L}$ (Arons & Scharlemann 1979), where θ_p is the angular width of the polar cap, θ_f is the angular width between the magnetic axis to the upper boundary field lines of the outer gap and r_f is the radial distance to the upper boundary field lines intercepting with the light cylinder. Zhang & Cheng (1997) has discussed the possible value of x_{max}. In general the γ-ray beaming solid angle should be different for various γ-ray pulsars, which is a function of the magnetic inclination angle as well as the size of the outer gap(Zhang et al. 2004). Some approximate forms of beaming solid angle have been given(Yadigaroglu & Romani 1995; Zhang, Zhang & Cheng 2000). How accurate of these approximation forms are not known. For simplicity, in this paper, we will treat these two parameters, $\Delta\Omega$ and x_{max} as constants. By fitting the γ-ray spectrum of known γ-ray pulsars, these two parameters are chosen to be $\Delta\Omega \sim 1sr$ and $x_{max} \sim 2$ respectively (Cheng & Zhang 1998).

In order to compare with the EGRET data, the integral flux is necessary and is given by

$$F(\geq 100MeV) = \int_{100MeV}^{E_{max}} F(E_\gamma) dE_\gamma \tag{6}$$

where E_{max} is the maximum energy of gamma-rays and is chosen to be 100GeV.

In Fig. 1 - Fig. 3, the model γ-ray spectra are calculated for various periods, magnetic fields and inclination angles respectively. We can see that the spectra become softer when the period decreases, the magnetic field increases or the inclination angle decreases in the energy range of EGRET (100MeV-10GeV). Alternatively speaking the spectral break becomes larger when the period increases, the magnetic field decreases or the inclination angle increases. We can understand these trends by examining the behavior of the energy break at Eq. 1. Typically most power radiated from the primary electrons/positrons come from regions with characteristic distance of order of $\sim R_L$. Substituting Eq. 3 and Eq. 4 into Eq. 1, we obtain the spectral break is $\propto \eta^{3/2} P^{3/28} B_{12}^{-3/28}$, which explains the relations between spectral variations in Fig. 1-3. However, in subsequent sections we will find that the key differences between γ-ray pulsars in the galactic plane and those in the high galactic latitude are: (1)galactic plane γ-ray pulsars are younger, shorter in period and have larger inclination angles and (2)they satisfy different relation between L_γ and L_{sd}. In fact, most high latitude γ-ray pulsars satisfy $L_\gamma \propto L_{sd}$ which means $f \sim 1$ (cf. Fig. 6f and the lower panel of Fig. 7). In this case, the spectral break is $\propto \eta^{21/16} P^{-1/8}$. Since galactic γ-ray pulsars have larger inclination angles and shorter periods in comparing with the high galactic latitude γ-ray pulsars, so the spectrum of γ-ray pulsars in high galactic latitude has a softer spectrum than those in the galactic plane. In Fig. 4, we compare the typical spectra between the high and low galactic latitude γ-ray pulsars.

4. Monte Carlo Simulation of γ-ray Pulsars in the Galaxy and in the Gould Belt

In order to consider the γ-ray luminosity and spatial evolution of pulsars in the Galaxy and in Gould Belt, the initial values of parameters of pulsar at birth, which include the initial position, velocity, period and magnetic field strength, are needed. The procedure of Monte Carlo method and the evolution of pulsar parameters are described in our previous works (Cheng & Zhang 1998; Zhang & Cheng 1999; Zhang, Zhang & Cheng 2000; Fan, Cheng & Manchester 2001; Zhang et al. 2004). Here, we briefly describe basic assumptions for generating the γ-ray pulsars (radio-quiet and radio-loud) in the Galaxy as well as in the Gould Belt:

1 The pulsars are born at a rate ($\dot{N}_{NS} \sim 1$ per century) in the Galaxy. The age of the Gould Belt is estimated to be ∼30 Myr old and the pulsars are born at a rate of ∼ 20 Myr^{-1} (Grenier 2000).

2 The Gould Belt has an ellipsoidal shaped ring with semi-major and minor axes equal to 500pc and 340pc respectively. The Sun is displaced from the center of Gould Belt about 200pc towards $l = 130°$(Guillout et al. 1998). On the other hand, there are other possible interpretations of the geometry of the Gould Belt. For examples, Olano (1982) and Moreno et al. (1999) have given smaller size for the Gould Belt. However, we will show that most γ-ray pulsars originated from the Gould Belt are mature pulsars with age nearly 1 million years. They already move very far away from the Gould Belt. The most important point is our solar system is enclosed in the Gould Belt, the exact dimensions of the Gould Belt is not very crucial in our problem.

3 The initial position for each pulsar in the Galaxy is estimated from the distributions $\rho_z(z) = (1/z_{\rm exp}){\rm exp}(-|{\rm z}|/{\rm z}_{\rm exp})$ and $\rho_R(R) = (a_R/R^2_{\rm exp})\ R\ {\rm exp}(-{\rm R}/{\rm R}_{\rm exp})$, where z is the distance from the Galactic plane, R is the distance from the Galactic center, $z_{exp} = 75$ pc, $a_R = [1 - e^{-R_{\rm max}/R_{\rm exp}}(1 + R_{\rm max}/R_{\rm exp})]^{-1}$, $R_{\rm exp} = 4.5$ kpc and $R_{\rm max} = 20$ kpc (Paczynski 1990; Sturner & Dermer 1996). But the initial position of each pulsar is assumed to be born uniformly inside the Gould Belt.

4 The initial magnetic fields are distributed as a Gaussian in $logB$ with a mean value of 12.5 and a dispersion of 0.3. Since the majority γ-ray pulsars are younger than 3 million years old and the field does not decay in 10 Myr (Bhattacharya et al. 1992). So we ignore any field decay for these rotation-powered pulsars.

5 The initial period is chosen to be P_0=10ms and the period at time t is Given by $P(t) = (P_0 + 1.95 \times 10^{-39} B^2 t)^{1/2}$. We would like to remark that the initial period is not very crucial for our problem because the typical age of γ-ray pulsars is of order of million years old.

6 The initial velocity of each pulsar is the vector sum of the circular rotation velocity at the birth location and random velocity from the supernova explosion(Paczynski 1990). The circular velocity is determined by Galactic gravitational potential and the random velocity is distributed as a Maxwellian distribution with a dispersion of three dimensional velocity $= \sqrt{3} \times 100$km/s (Lorimer et al. 1997). Furthermore, the pulsar position at time t is determined following its motion in the Galactic gravitational potential. Using the equations given by Paczynski (1990) for given initial velocity, the orbit integrations are performed by using the 4th order Runge Kutta method with

variable time step (Press et al. 1992) on the variables R, V_R, z, V_Z and ϕ. Then the sky position and the distance of the simulated pulsar can be calculated.

7 The inclination angle (α) of each pulsar is chosen randomly from a uniform distribution (Biggs 1990).

8 The following radio selection effects are used. The pulsar must satisfy that its radio flux is greater than the radio survey flux threshold and its broadened pulse width is less than the rotation period (e.g. Sturner & Dermer 1996). **We calculate the 400 MHz radio luminosity, L_{400}, of each model pulsar using the following distribution given by (Narayan & Ostriker 1990) $\rho_{L_{400}}(P, \dot{P}) = 0.5\lambda^2 e^{-\lambda}$, where $\lambda = 3.6(\log(L_{400}/ < L_{400} >) + 1.8)$, $\log < L_{400} >= 6.64 + (1/3)\log(\dot{P}/P^3)$, and L_{400} is in units of mJy kpc^2. The minimum detectable average flux density, S_{min}, of a pulsar's radio survey is estimated by using the method of Sturner & Dermer (1996) (also see Cheng & Zhang 1998)**. The pulsar which satisfies $L_{400}/d^2 \geq S_{\min}$ is considered to be a radio-detectable pulsar, where L_{400} is the radio luminosity at 400 MHz and d is the distance to the pulsar. The radio beaming fraction can be expressed as (Emmering & Chevalier 1989) $f_r(\omega) = (1-\cos\omega)+(\pi/2-\omega)\sin\omega$, where $\omega = 6^{\circ}.2 \times P^{-1/2}$ (e.g. Biggs, 1990) is the half-angle of the radio emission cone. Then, following Emmering & Chevalier (1989), a sample pulsar with a given period P is chosen in one out of $f_r(P)^{-1}$ cases using the Monte Carlo method. **There are growing evidence that very strong surface magnetic field exists on the surface of neutron stars. For examples, absorption/emission line features have been observed in isolated pulsars/neutron stars,e.g. 1E 1207.4-5209 (Sanwal et al. 2002), PSR 1821-24 (Becker et al. 2003) and RBS1223 (Haberl et al. 2003). These line features imply that the surface magnetic fields are one to two order of magnitude higher than that of the dipolar field. In fact, Cheng & Zhang (1999) have analyzed the X-ray emission from the polar cap regions of the rotation-powered pulsars and concluded that there exists a characteristic surface field with strength $\sim 10^{13}$G. Such strong surface magnetic field can easily affect the beaming direction of radio wave.** Therefore, we assume that the beaming direction of γ-rays is independent of the beaming direction of the radio. In general, it is possible that there is only one beaming, either the radio or the gamma-ray beam, pointing toward us. In fact this is one of key differences between outer gap model and polar gap model. The solid angle of γ-rays is taken to be $1sr$.

9 The γ-ray threshold varies over the sky. Yadigaroglu & Romani (1995) used a flux threshold of 3×10^{-10} erg cm^{-2}s^{-1}, which can compare to the faintest 5σ sources in

the first EGRET catalog(Fichtel et al. 1994). However, in the third EGRET catalog, the faintest source in the catalog with significance $\sqrt{TS}$ $\geq$4 has a photon flux of $(6.2 \pm 1.7) \times 10^{-8}$ cm^{-2}s^{-1} (Hartman et al. 1999). Gonthier et al. (2002) have argued that this threshold could be reduced for $\mid b \mid > 10^{\circ}$. In our analysis, we include the criterion of the likelihood $\sqrt{TS}$ $\geq$5 ($\sim 5\sigma$) which corresponds to the energy threshold of $S^{th}_{\gamma}(E_{\gamma} > 100MeV) \geq 1.2 \times 10^{-10}$ erg cm^{-2}s^{-1} for $\mid b \mid < 10^{\circ}$ but decreases the threshold to $S^{th}_{\gamma}(E_{\gamma} > 100MeV) \geq 7.0 \times 10^{-11}$ erg cm^{-2}s^{-1} for $\mid b \mid > 10^{\circ}$ (Gonthier et al. 2002).

5. Simulation results and Discussion

Following the procedure described in section 4, we perform Monte Carlo simulation of Galactic pulsars born during past 3×10^7 yrs. We use the code of Cheng & Zhang (1998) (also see Zhang et al. 2000) in our simulation, in which Parkes Multibeam survey is not included. We summarize the various components of simulated γ-ray pulsars in table 1. Our results suggest that there should be more unidentified EGRET γ-ray sources identified as radio pulsars. In particular, the Parkes multi-beam pulsar survey is expected to discovery more radio pulsars (Manchester et al. 2001). Torres et al. (2001) have found five new possible radio pulsar-Unidentified source associations. Most recently, Kramer et al. (2003) have used the newly release survey results to correlate the unidentified EGRET γ-ray sources and, have found more than 35 coincidences and around 20 probable associations. It is important to note that the number in each box of table 1 is sensitive to the input parameters, e.g. detection threshold, birth rate, initial distributions of pulsars etc., for a given theoretical model. However, their ratios are less sensitive to most input parameters except their relative birth rate. It is interested to note that the γ-ray pulsars in the Gould Belt contribute significant fraction of total γ-ray pulsars for $\mid b \mid > 5^{\circ}$ and become unimportant for $\mid b \mid < 5^{\circ}$. This is simply because the solar system is enclosed by the Gould Belt.

In Fig. 5a-e, we compare our simulated distributions of period, period derivative, magnetic field, distance and energy flux with the observed data of 8 known radio-loud γ-ray pulsars (i.e. Crab, Vela, Geminga, PSR 0656+14, PSR 1046-58, PSR 1055-52, PSR 1509-58, PSR 1706-44, PSR 1951+32) using KS test. Although PSR 0656+14 and PSR 1509-58 are confirmed in γ-ray band, there are insufficient γ-ray photons to provide the information of pulsed fraction. So we did not include these pulsars in the cumulative plot of energy flux. The maximum deviations of period, period derivative, magnetic field, distance and γ-ray energy flux distributions from the observed distributions are 0.36, 0.32, 0.25, 0.33 and 0.21

respectively. It can be seen that four of five accumulative distributions cannot be rejected at better 80% confidence level, and period cumulative distribution cannot be rejected at better than 90% confidence level. Therefore we conclude that the model results do not conflict with the observed data of γ-ray pulsars.

In Fig. 6a-f, we plot the normalized distributions of period, magnetic field, age, distance, inclination angle and the fractional size of outer gap. The solid lines and the dashed lines are γ-ray pulsars located at $\mid b \mid < 5^\circ$ and $\mid b \mid > 5^\circ$ respectively. We can clearly see that there are two classes of γ-ray pulsars. γ-ray pulsars located at $\mid b \mid < 5^\circ$ have shorter periods, younger, larger inclination angles and smaller outer gap size in comparing with γ-ray pulsars located at $\mid b \mid > 5^\circ$. As we have mentioned that γ-ray pulsars in high galactic latitude are dominated by mature pulsars, which are old enough to move up the high latitude and to evolve to longer periods. Young γ-ray pulsars will be dominated in the galactic plane, which have stronger γ-ray luminosity, larger inclination angle, shorter period and larger magnetic field. In Fig. 6, γ-ray pulsars in high galactic latitude are actually closer than those in galactic plane. It is because they are older and weaker γ-ray pulsars, so they must be near otherwise they cannot be detected.

In Fig. 7, we plot L_γ versus L_{sd}, where the upper panel is γ-ray pulsars located at $\mid b \mid < 5^\circ$ and lower panel is γ-ray pulsars located at $\mid b \mid > 5^\circ$. We can see that there are two distinctive regions. For $L_{sd} > 3 \times 10^{34}$ erg/s, the relation between L_γ and L_{sd} is rather scattered. However, for $L_{sd} < 3 \times 10^{34}$ erg/s, L_γ is proportional to L_{sd}. In particular, the γ-ray pulsars in high latitude, most pulsars satisfy $L_\gamma \propto L_{sd}$, which means $f \sim 1$. This is supported by Fig. 6f, in which the fractional sizes of outer gap f for γ-ray pulsars located at $\mid b \mid > 5^\circ$ are all close to unity whereas the fractional sizes of outer gap for γ-ray pulsars located at $\mid b \mid < 5^\circ$ have a wide distribution. If $L_\gamma \propto L_{sd}^\beta$ is used to fit all γ-ray pulsars, then $\beta \sim 0.3$ for γ-ray pulsars located at $\mid b \mid < 5^\circ$ and $\beta \sim 0.6$ for γ-ray pulsars located at $\mid b \mid > 5^\circ$ respectively. Since the outer gap size is a function of $B, P and \alpha$. For young pulsars, there are more combination of these three parameters to make the outer gap size less than unity. On the other hand, for old γ-ray pulsars their periods are already so long that there are not much room for other two parameters to vary the outer gap size away from unity. The spatial distributions of γ-ray pulsars are given in Fig. 8.

In Fig. 9a-d, we compare distributions of γ-ray pulsars from the Galaxy (dotted lines) and from the Gould Belt (solid lines). We can see that the properties of γ-ray pulsars from two sources are very similar except γ-ray pulsars from the Gould Belt have lower population and closer. These two properties results from the lower birth rate in the Gould Belt and the solar system is inside the Gould Belt.

6. Conclusion and discussion

We have studied the γ-ray emission properties of pulsars in the Galaxy as well as in the Gould Belt by using a new self-consistent outer gap model, which includes the effects of inclination angle and average properties of outer gap. We have found that this new model can produce more mature γ-ray pulsars, which have longer period, smaller inclination angle and typical age of one million years old, than the old model. In fact the mature γ-ray pulsars dominate in the high galactic latitude. The spectra of these mature pulsars are significantly softer and weaker than the young pulsars in the galactic plane. It is because the spectral hardness is roughly determined by the typical photon energy given in equation (1), which characterizes the position of spectral break. The explicit form of how the typical photon energy depends on the inclination angle is given in equations (28) and (38) of Zhang et al. (2004). Although the form is quite complicated, we can roughly understand why smaller inclination angle gives smaller typical photon energy and hence **softer** spectrum by equation (1). It is because the typical position of the outer gap is the null surface where becomes larger for smaller inclination angle. From the simulation results, it is not clear why γ-ray pulsars in high latitude should have smaller inclination angle. It is because all of these high latitude pulsars are older pulsars, which have longer period. Without the compensation of the effect of inclination angle, the fractional size of the outer gap f is larger than unity (cf. equation 3). In order to maintain the outer gap size less than unity, η in equation (4), which contains the effect of inclination angle must be small. In fact η becomes small for small inclination angle (cf. Fig. 1 of Zhang et al. 2004). Therefore high latitude gamma-ray pulsars tend to have smaller inclination angle and harder spectrum. On the other, this seems to imply that the confirmed radio-loud γ-ray pulsars in the galactic plane should have larger inclination. This statistical correlation has not yet been established. But we would like to point out that the values of inclination angle of pulsars are very difficult to be measured accurately and there are only 7 confirmed γ-ray pulsars. So it is very difficult to find such correlation in radio-loud γ-ray pulsars. However, we do believe that this correlation should be there when more radio-loud γ-ray pulsars are discovered in the galactic plane.

We have also used a Monte Carlo method to simulate the properties of the γ-ray pulsar population in the Galaxy as well as in the nearby Gould Belt in terms of the revised outer gap models. The initial magnetic field, spatial and velocity distributions of the neutron star at birth which are obtained by the radio pulsar statistical studies have been used in our simulations. We have obtained the spatial, distance, period, age, magnetic field, inclination angle, photon flux distributions of the radio-loud and radio-quiet γ-ray pulsars. We find that the properties of γ-ray pulsars between $\mid b \mid < 5^\circ$ and $\mid b \mid > 5^\circ$ are very much different. Galactic plane γ-ray pulsars are younger, shorter in period and have larger inclination angles. They satisfy different relation between L_γ and L_{sd}. The high latitude γ-ray pulsars satisfy

$L_\gamma \propto L_{sd}^{0.6}$ but the galactic plane pulsars satisfy $L_\gamma \propto L_{sd}^{0.3}$.

The present model predicts very similar numbers of γ-ray pulsars as old models but many more high latitude γ-ray pulsars than old models (Cheng & Zhang 1998; Zhang, Zhang & Cheng 2000). It is because the new model allows the outer gap can exist about a few million years old for appropriate combination of B,P and α, so old γ-ray pulsars can move up to high galactic latitude. Furthermore, the nearby Gould Belt also contribute significant number of γ-ray pulsars in high latitude. Torres et al. (2003) have predicted that AGILE can detect more γ-ray pulsars (for a general review of AGILE cf. Tavani et al. 2001). Perhaps AGILE can provide some clues to differentiate polar gap model predictions (Gonthier et al. 2002) and outer gap model predictions, and GLAST makes the final verdict. Finally we would like to remark that the better predictions on how many γ-ray pulsars should be detected by AGILE and GLAST should include a better expression of γ-ray solid angle, which depends on the inclination angle as well as the outer gap size. However, in order to have a reliable expression of γ-ray solid angle, we need to carry out three-dimensional model calculation and come up with an approximate expression from Monte Carlo simulation.

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Table 1: Various Components of Simulated γ-ray Pulsars

	Gould Belt		Galaxy	
Birth rate	1/50,000yr.		1/100yr.	
Galactic latitude $\|b\|$	$< 5°$	$> 5°$	$< 5°$	$> 5°$
Radio-loud	1	4	12	4
Radio-quiet	2	13	40	22

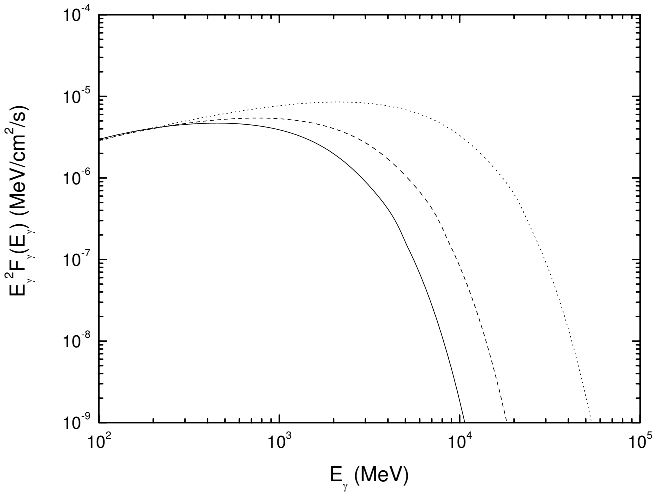


Fig. 1 Distribution of the energy flux vs energy of the photon. In this figure, B_{12} has been taken to be 3, P to be 0.4 s and α to be 20° (solid line), 40° (dashed line) and 60° (dotted line).

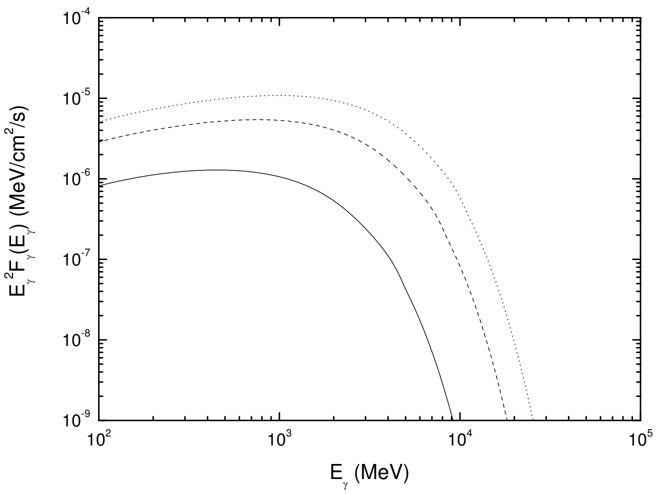


Fig. 2 Distribution of the energy flux vs energy of the photon. In this figure, P has taken to be 0.4 s, α to be 40° and B_{12} to be 1 (solid line), 3 (dashed line) and 5 (dotted line).

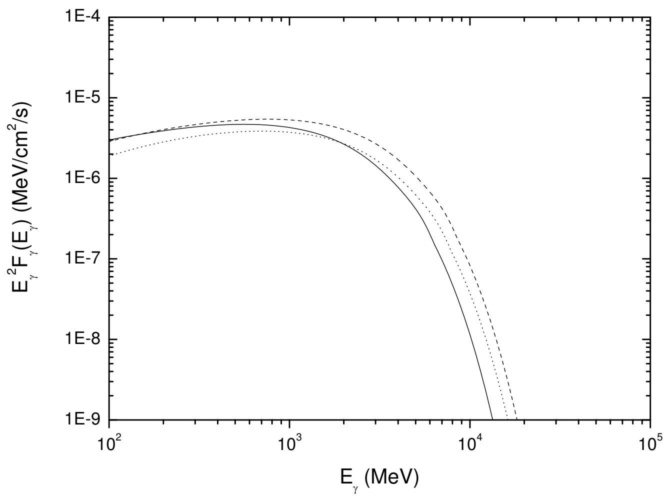


Fig. 3 Distribution of the energy flux vs energy of the photon. In this figure, B_{12} has taken to be 3, α to be 40° and P to be 0.2 s (solid line), 0.4 s (dashed line) and 0.6 s (dotted line).

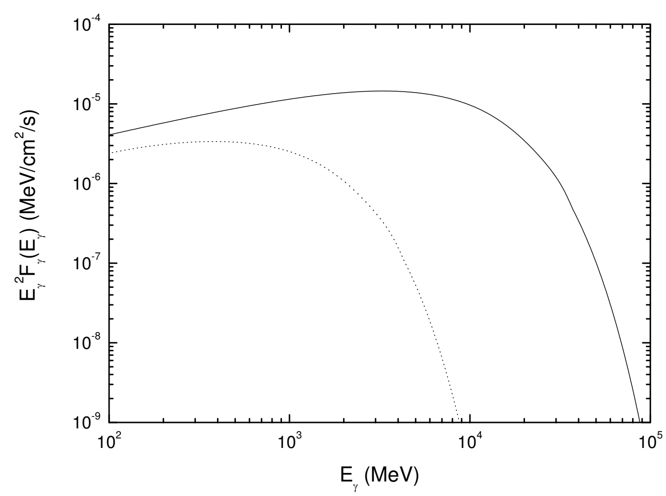


Fig. 4 Distribution of the energy flux vs energy of the photon. The solid line is the typical photon spectrum of a galactic γ-ray pulsar with $B_{12} = 5$, $P = 0.1s$ and $\alpha = 70°$, and the dashed line is the typical photon spectrum of a γ-ray pulsar in high galactic latitude with $B_{12} = 2$, $P = 0.3s$ and $\alpha = 20°$ respectively.

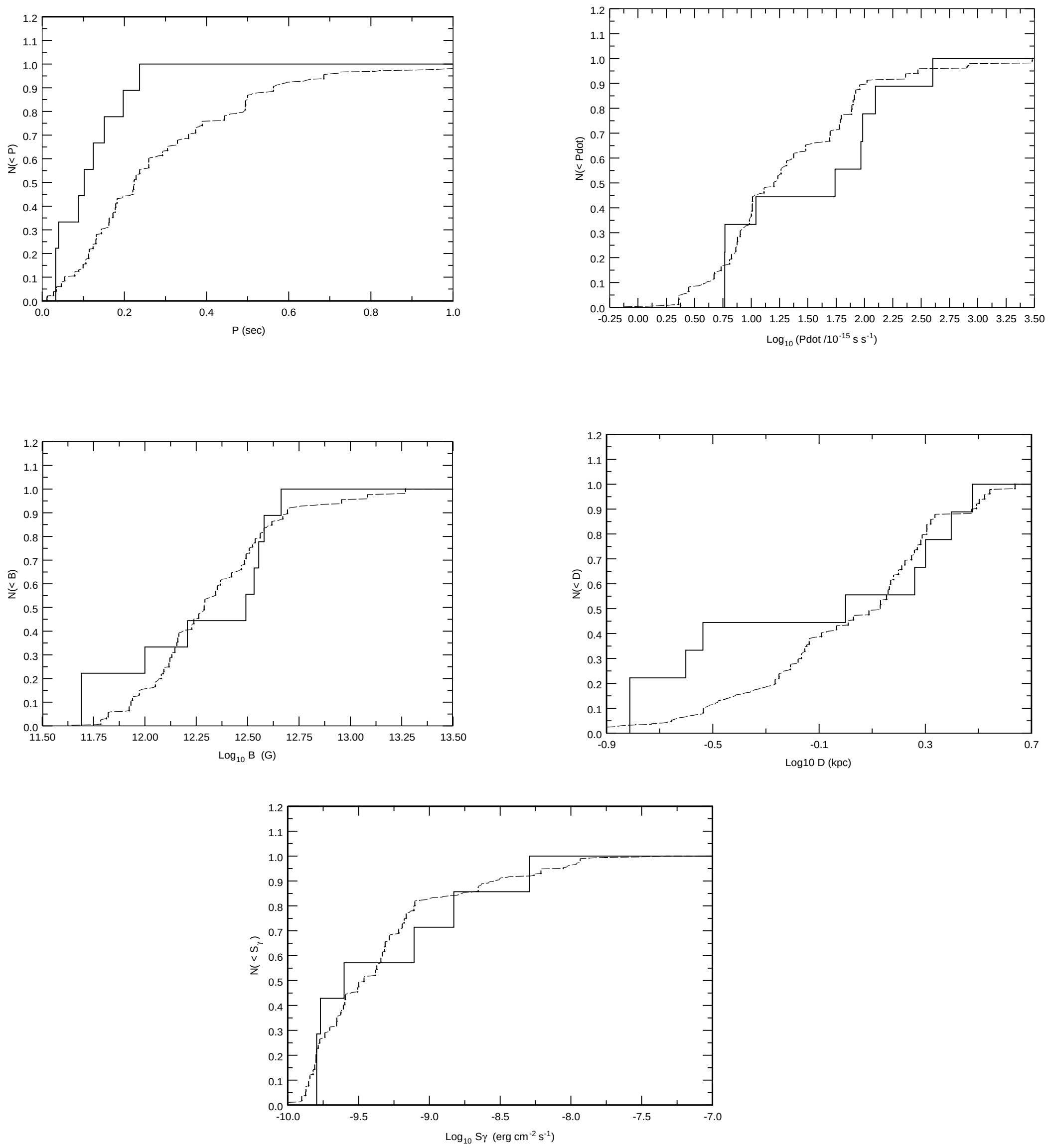


Fig. 5 Normalized cumulative distributions (**dashed curves**) of period, period derivative, magnetic field, distance and γ-ray energy flux of γ-ray pulsar population for our model with radio selection effects. For comparison, corresponding distributions (**solid lines**) of the observed γ-ray pulsars are also shown.

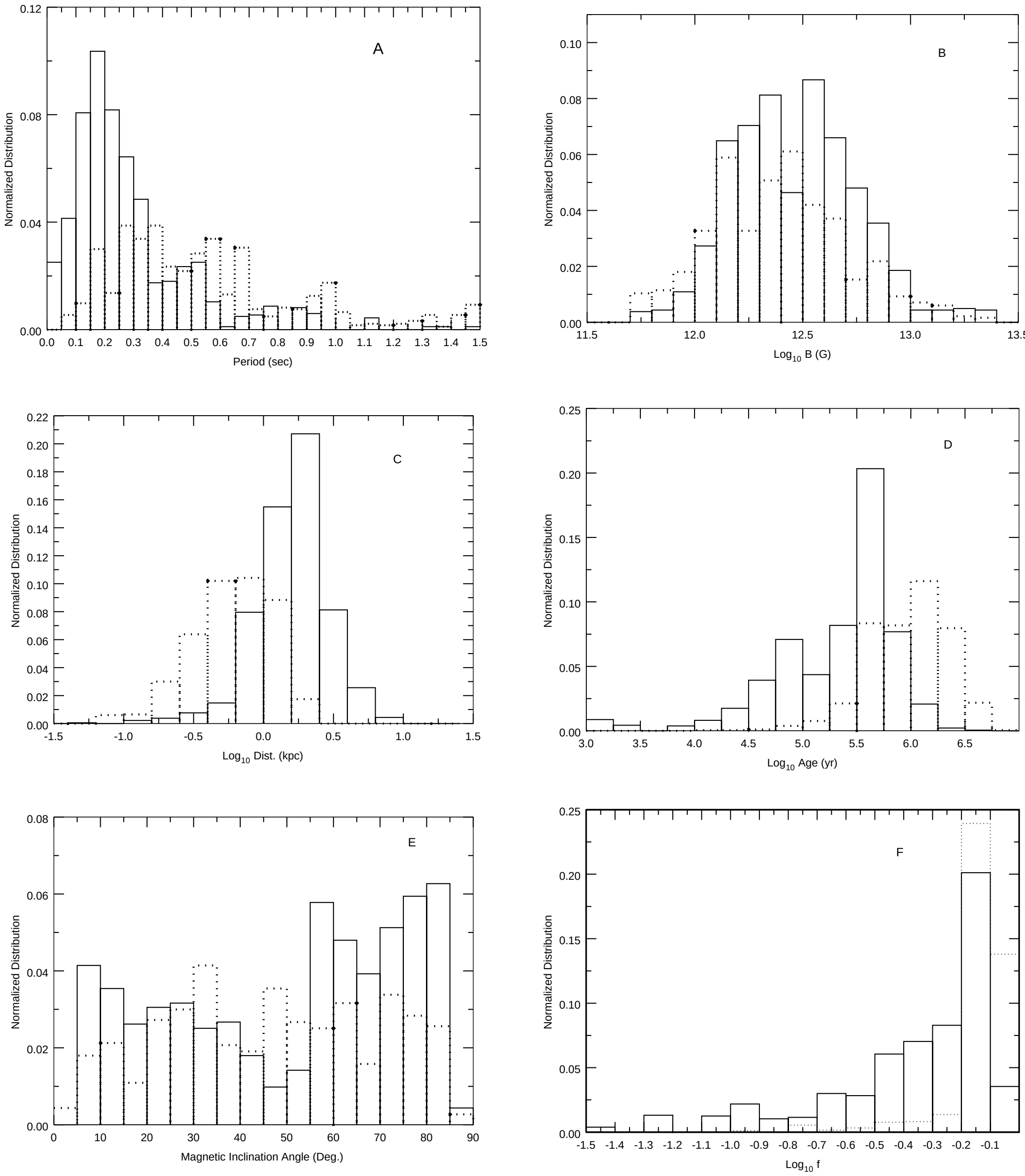
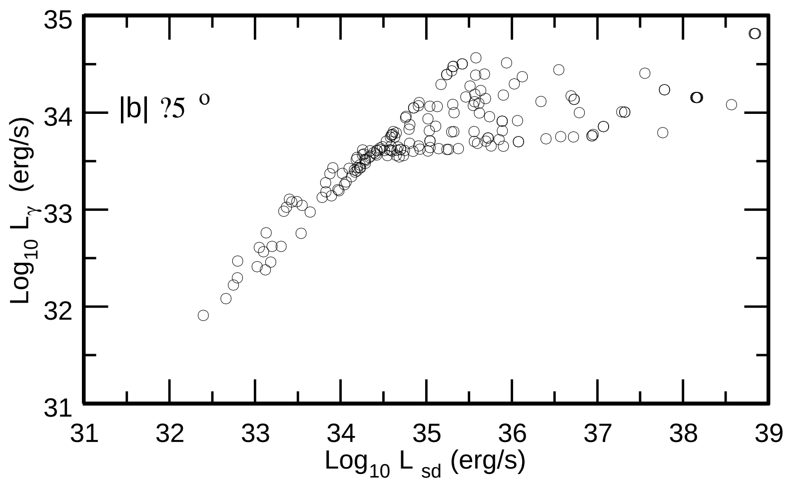


Fig. 6 Normalized cumulative distributions of period, magnetic field, distance, age, inclination angle and the averaged fractional size of outer gap of both simulated γ-ray pulsars in $|b| \leq 5^\circ$ (solid lines)and in $|b| > 5^\circ$ (dashed lines), **which are labelled by A, B, C, D, E, F respectively**.



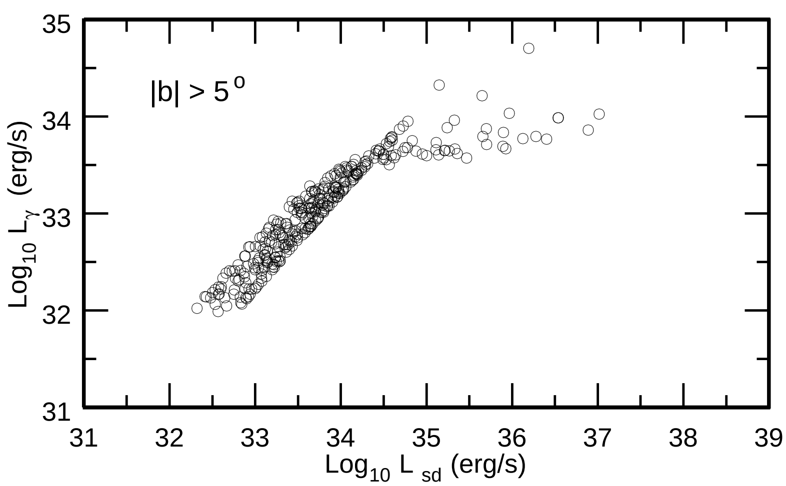


Fig. 7 Plot of γ-ray luminosity vs. spin-down luminosity for the simulated γ-ray pulsars. Upper panel for the case in $|b| \leq 5^{\circ}$, and bottom panel for the case in $|b| > 5^{\circ}$.

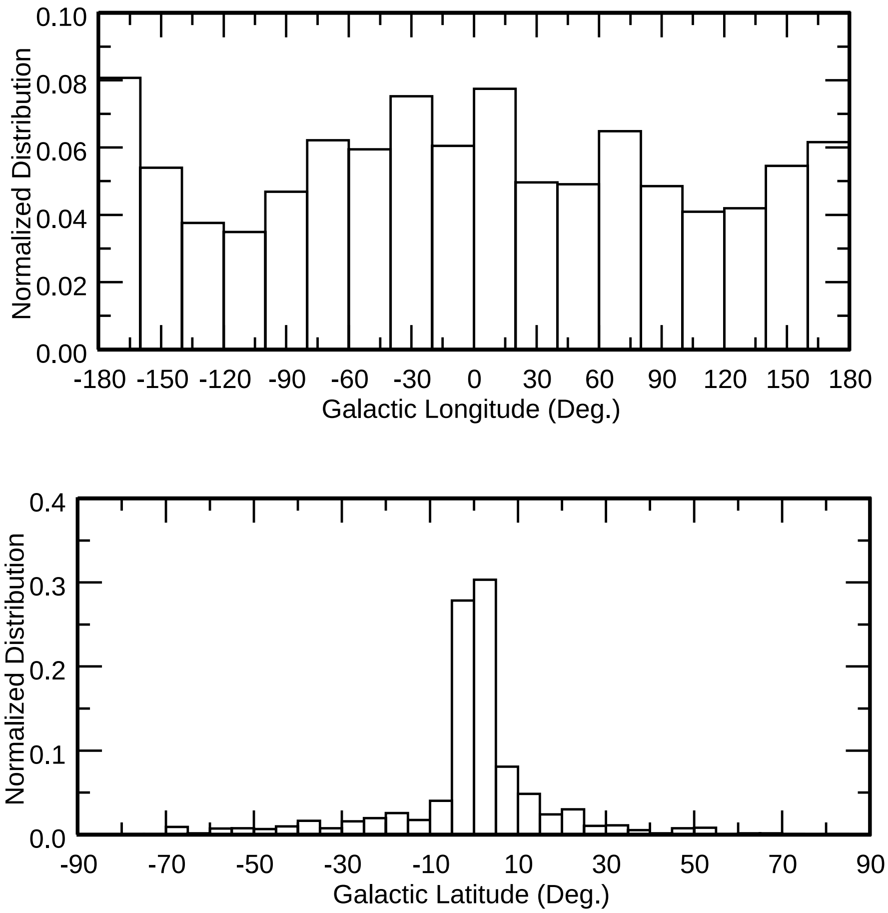


Fig. 8 Normalized distributions of the simulated γ-ray pulsars in Galactic longitude and latitude.

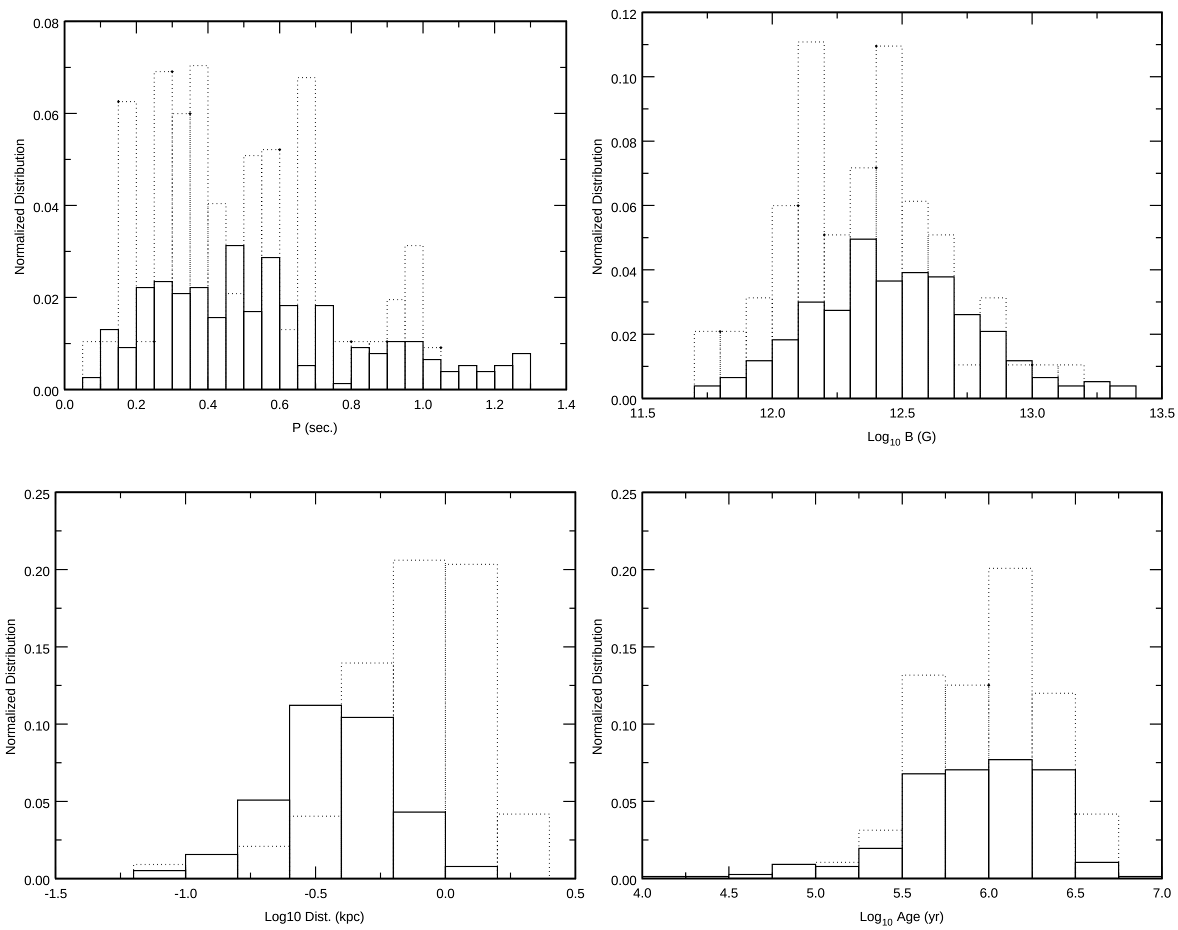


Fig. 9 Normalized distributions of period, magnetic field, distance and age of the simulated γ-ray pulsars for $|b| > 5^\circ$. Dotted histograms represent the distributions of the simulated γ-ray pulsars without those produced in the Gould belt, and solid histograms represent the simulated γ-ray pulsars only produced in the Gould belt.